Wave-front control and simulation for ExAOC

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ExAOC Midterm Review

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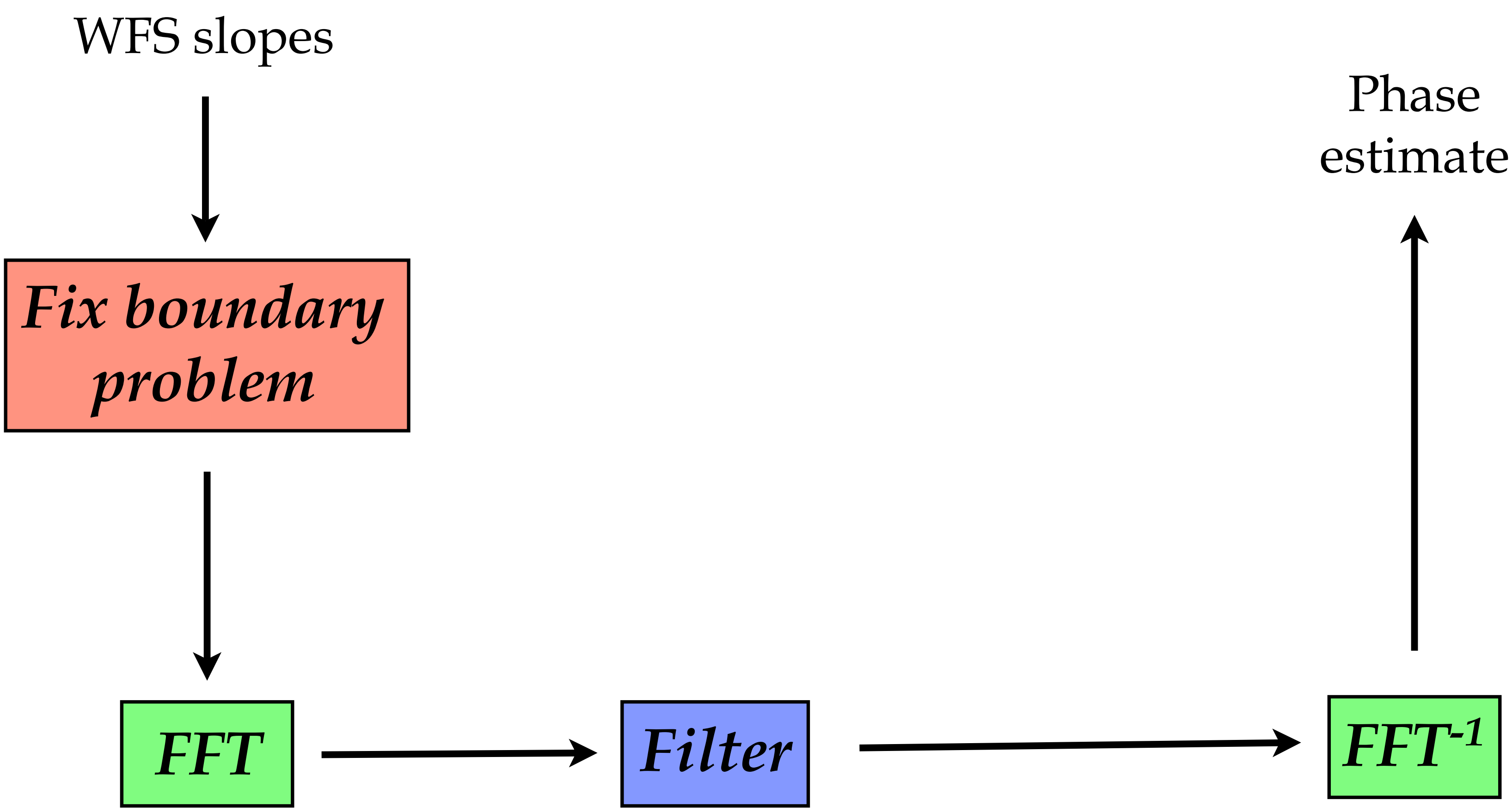
ExAOC tasks for wave-front control

- Where were we last spring?
  - *Optimal modal control with VMM works in Altair, but much too expensive for ExAOC*
  - *Fourier Transform Reconstruction (FTR) algorithm efficient enough for ExAOC and validated, but not adaptive to operating conditions*
- What have we done?
  - *We've combined the best of both methods, with a couple of bonuses*
  - *We have a detailed end-to-end simulation of the AO system to contribute to system design and performance analysis*

Optimal Modal FTR is our solution

- Modal set is the Fourier basis. This works even on an arbitrary aperture.
  - *We can create a 'modal' filter customized to the AO system*
- Reconstruction at each time step is with FTR.
- Closed-loop modal coefficients are used to estimate optimal gains for control law for each mode. Gains are implemented as a filter.
- Computationally feasible for 64x64 ExAO right now.
- Extra benefits include
  - *Modal coefficients are available for free, unlike matrix-based modal control, which requires extra computation.*
  - *There is a natural relationship between filter structure and PSF structure.*

FTR works by filtering



Modal numbering convention

Modal grid for 8 x 8 case

l \ k	0	1	2	3	4	-3	-2	-1
-1		57 58						
-2		45 46	59 60					
-3		35 36	47 48	61 62				
4	19	27 28	37 38	49 50	63			
3	11 12	20 21	29 30	39 40	51 52			
2	5 6	13 14	22 23	31 32	41 42	53 54		
1	1 2	7 8	15 16	24 25	33 34	43 44	55 56	
0	0	3 4	9 10	17 18	26			

- We only just over half of the pairs [k,l] due to Hermitian symmetry.
- We index the modes from piston to waffle
- All filters will be Hermitian

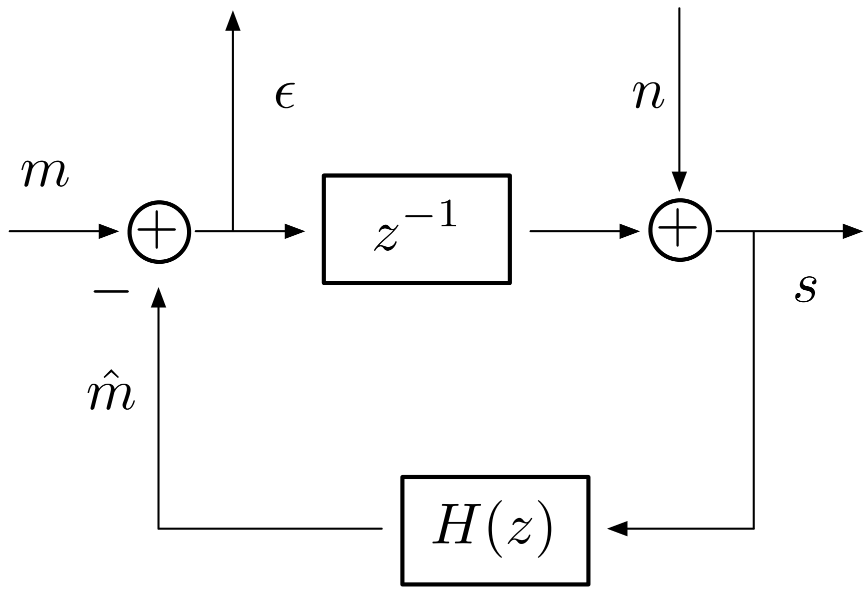
We can do all this with an aperture

- Fourier basis in an arbitrary aperture is a tight Frame that allows analysis and synthesis like an ONB.
- If we window the data, we can use a fast DFT to get modal coefficients.
- New method of slope management called edge correction ensures high-quality coefficient estimation by making outside region of phase flat.

- Result - we get the modal coefficients for free at each time step in the FTR process.

Optimal modal control scheme

- We follow Altair's implementation and assume an approximate model of control system (exact in simulation case) for each of the independent modes.
- We control a mode with feedback in the presence of noise.



Block diagram of control loop for a modal coefficient

Optimize for the minimum squared-residual error

- Since the noise at any step is independent of past errors, if we minimize on the measurement s, we minimize on the residual error.
- If we had perfect knowledge we would minimize

$$\mathcal{J} = \int \left| \frac{1}{1 + \exp(-j\omega)H(\omega)} \right|^2 [M(\omega) + N(\omega)]\ d\omega$$

- But we don't... so we have to estimate the open-loop PSD from the closed-loop measurements using

$$\hat{M}(\omega) + \hat{N}(\omega) = \left|1 + \exp(-j\omega)H_0(\omega)\right|^2 \hat{S}(\omega)$$

Gain estimation for FTR (1)

- From closed-loop telemetry, we estimate the closed-loop measurement PSDs
- Convert these to open-loop PSD estimates
- Find the control law which minimizes the error for the sine and cosine modes together

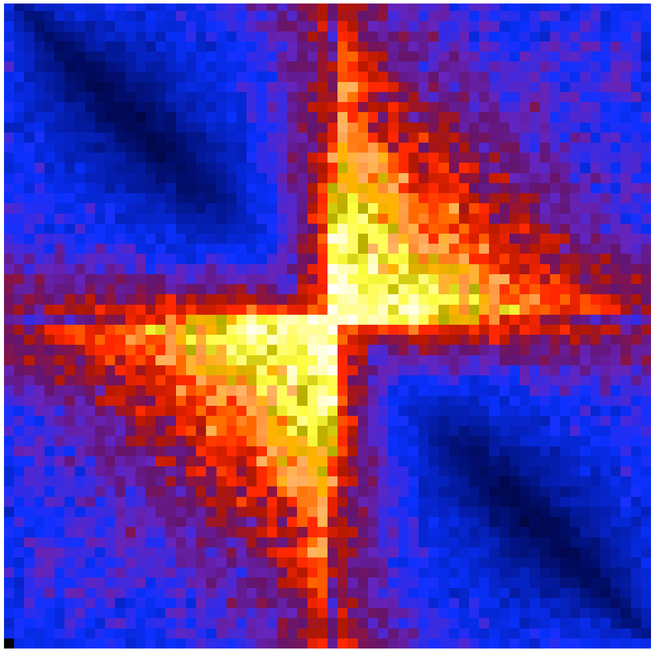
$$\operatorname{argmin} H(z) \left\{ \int \left| \frac{1}{1+\exp(-j\omega)H(\omega)} \right|^2 |1+\exp(-j\omega)H_0(\omega)|^2 \left[\hat{S}_S(\omega) + \hat{S}_C(\omega) \right] d\omega \right\}$$

- Where our control law is simple: $H(z) = \dfrac{g}{1 - cz^{-1}}$

Gain estimation for FTR (2)

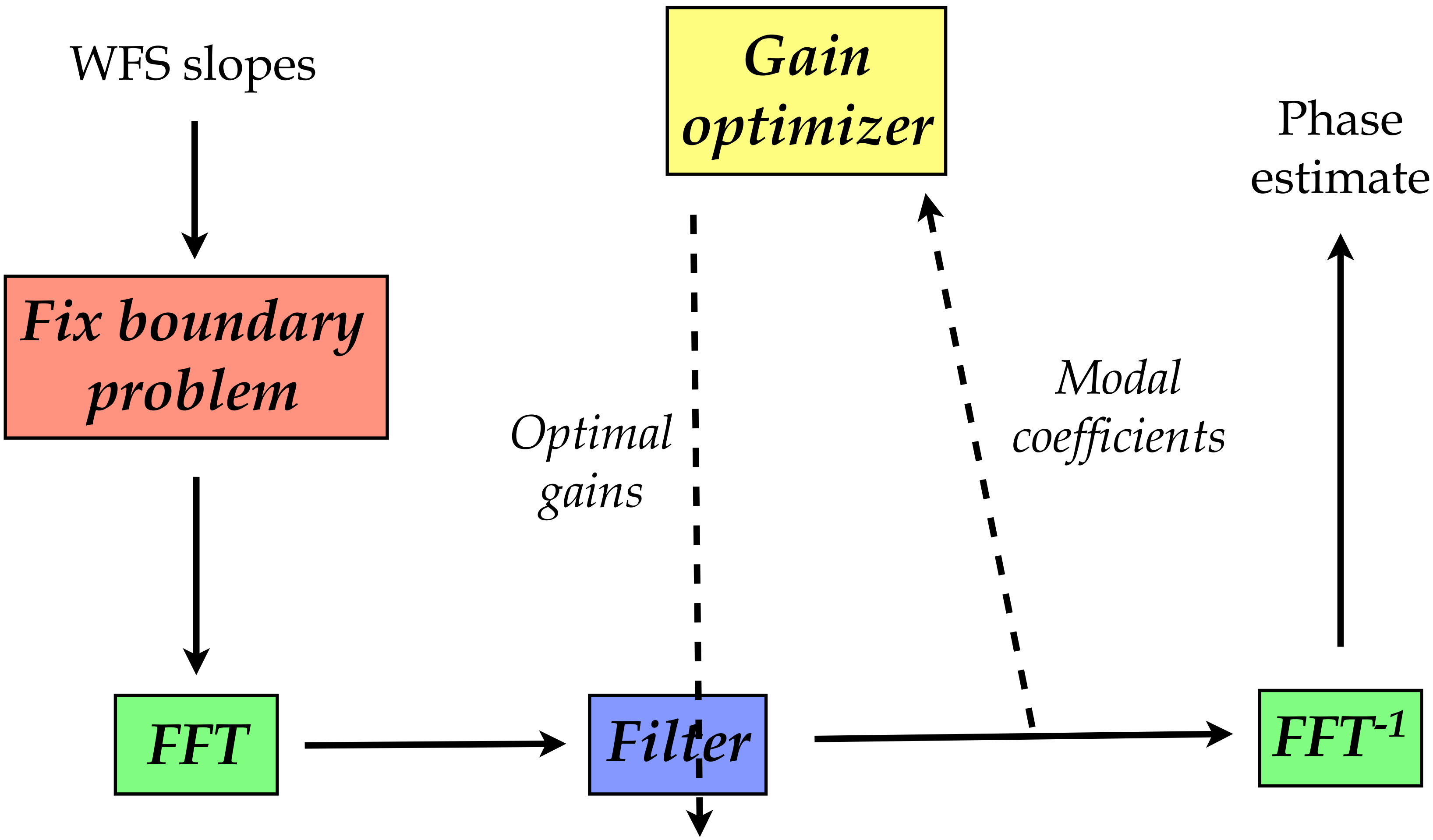
- For a single variable (gain g) we can solve the optimization problem efficiently.
- At each frequency [k,l] we have a gain - we construct the filter of these gains using Hermitian symmetry. This filter in then incorporated into the reconstruction filter.

Wind direction



Example filter, N=64

Gain optimization integrates into filter

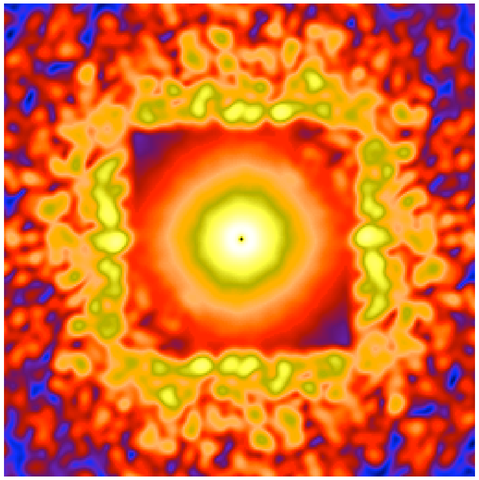


Gain optimization in incorporated into ExAOC end-to-end simulation

- Features of ExAOC simulation include:
  - *Fourier Optics for Spatially-filtered WFS onto CCD, quadcell config*
  - *Altair-based DM model using influence functions*
  - *Input phase aberration is a very long screen shifted at 10m/s,* r_0 *= 18 cm*
  - *FTR reconstruction at each time step*
  - *Modal coefficients obtained in reconstruction stage all steps, all modes*
  - *Gain optimization every 128 x 8 time steps (as describe above)*
  - *Full diagnostics including long-exposure PSDs and PSFs from the residual wavefront and instantaneous residual error in different spatial frequency bands*
  - *Run either single long case to watch optimization or many short cases with a specific filter to analyze general case performance*

We can control modes independently

- Given an optimal gain profile, we compare three filters
  - *(1) constant gain of 0.6 for all modes (2) optimized gains*
  - *(3) constant gains with optimized for a smaller region of filter*
- PSD of case (3) is almost exactly the combination of parts of the other two responses in the right places



Constant ***Mix*** ***Optimal***

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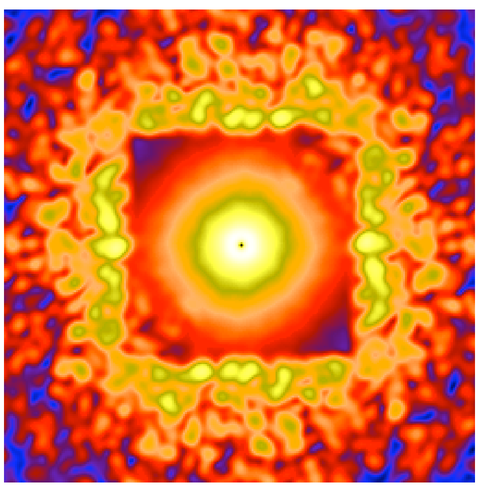
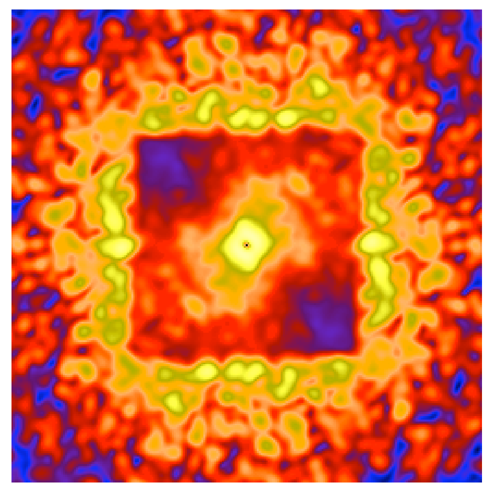


Constant

Mix



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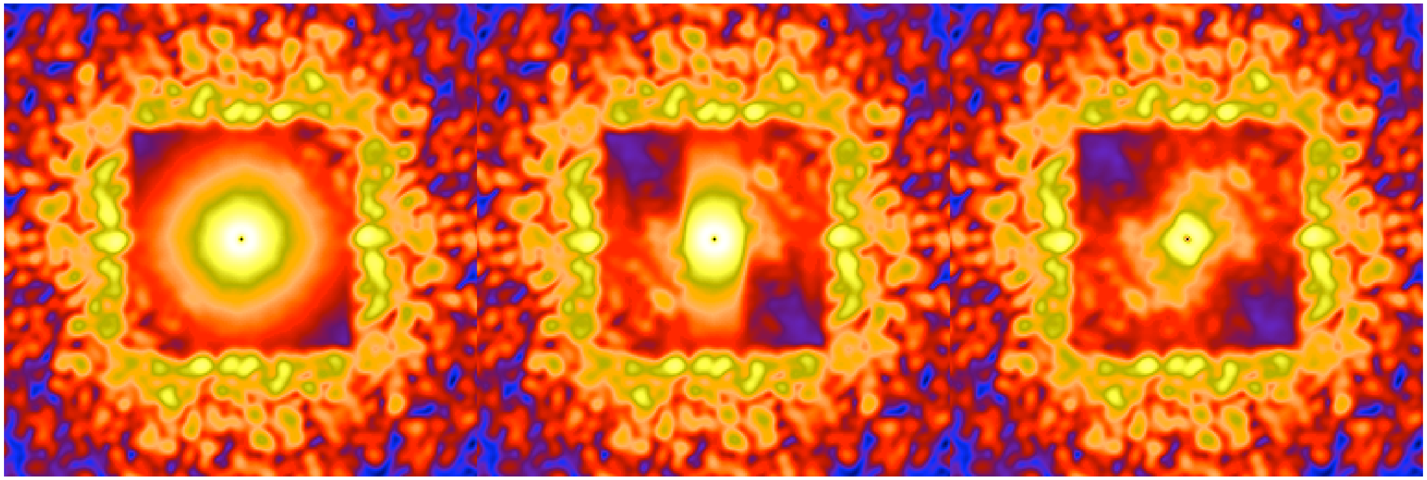
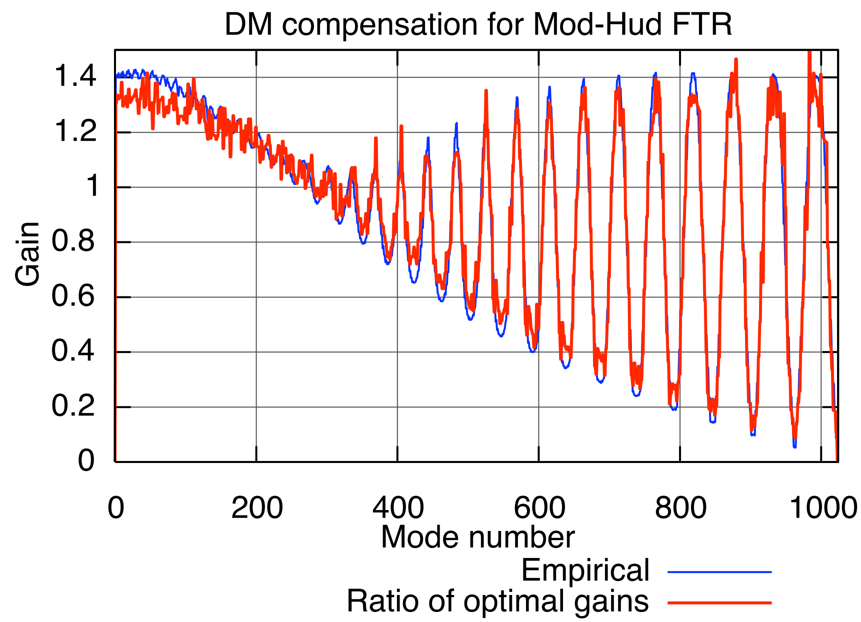


Constant ***Mix*** ***Optimal***

Gain optimization leads to DM compensation

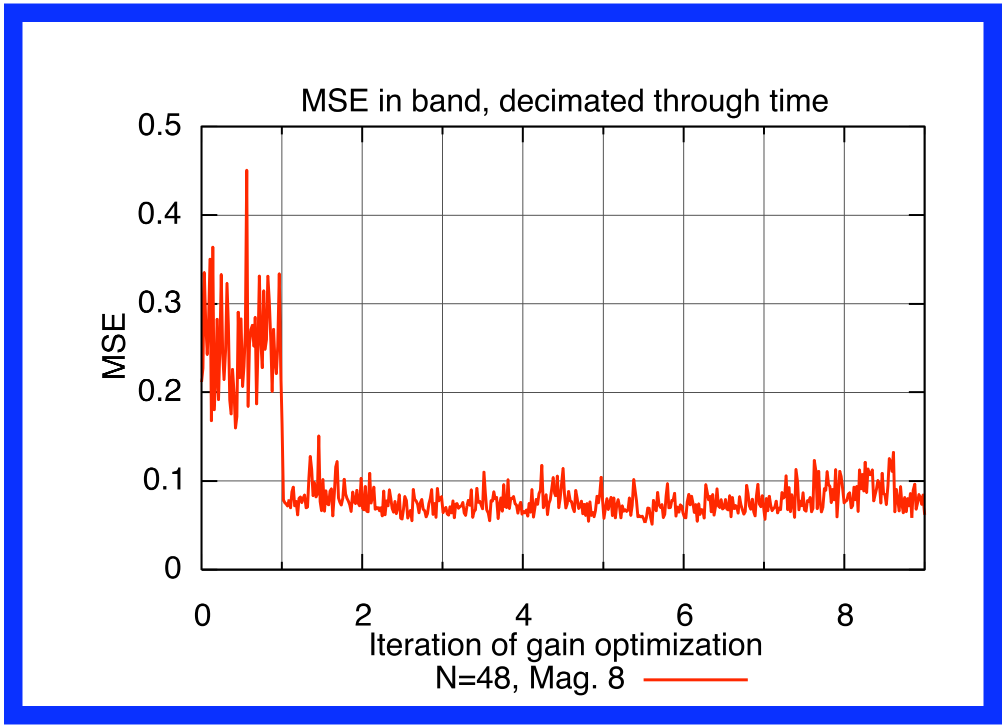


Data from 32x32 system run for 9 iterations of optimal gain estimation

- Hypothesis: gain estimation should compensate for Mod-Hud's lack of knowledge of DM
- Method: compare the ratio of the optimal gains for Mod-Hud and custom FTR on the same input

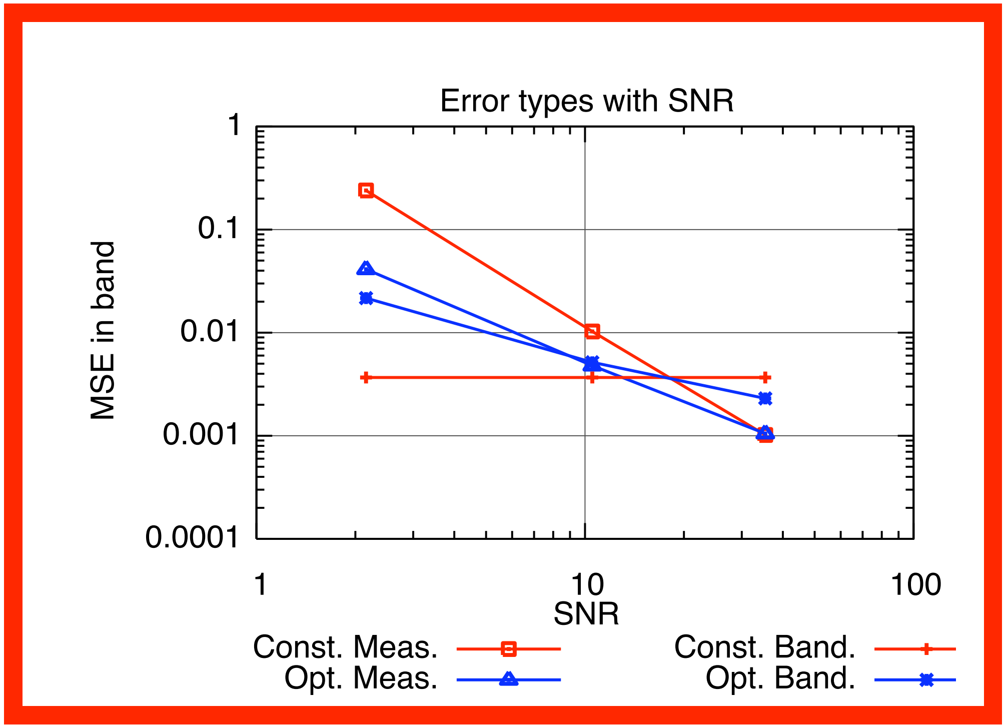
Optimal gains can significantly reduce residual wave-front error

- Use of optimal gains improves performance
  - *significant reduction in residual MSE at each timestep*
  - *less variation in MSE at each timestep*



N=48, NGS Mag 8 example for 8 iterations of gain optimization

Optimal gains trade-off bandwidth and measurement errors

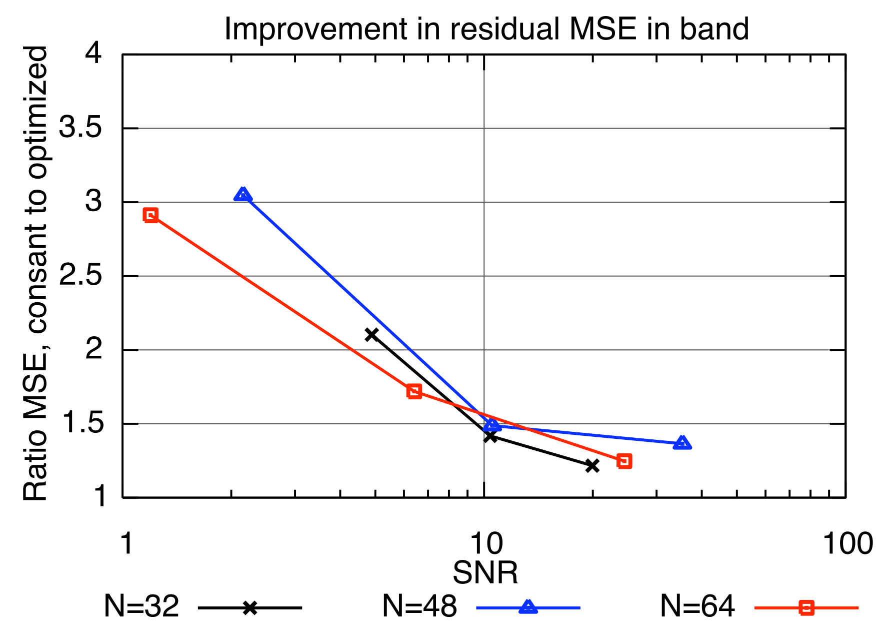


Data for N=48, median over a set of 25 random phase screens

- At high SNRs, optimal gains produce equivalent or more measurement error but less temporal error than before
- At low SNRs, optimal gains produce less measurement error but more temporal error than before

Biggest improvement due to optimization comes at low SNRs

- For all cases, optimized gains improved performance.
  - *At high SNRs, high spatial frequency bandwidth errors are reduced.*
  - *At low SNRs, measurement error is drastically reduced.*



Data points are medians from sets of random phase screens

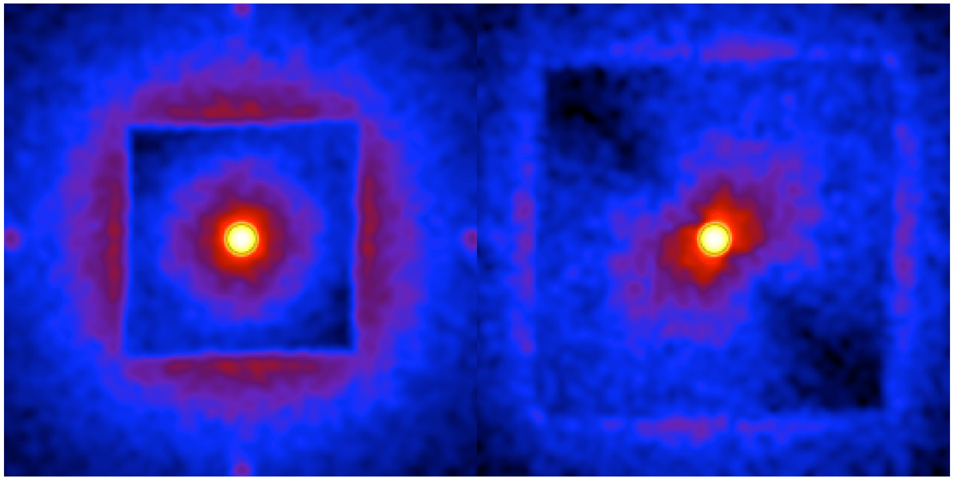
Gain optimization will be significant in system design

- In our simulations for NGS I=8, turning on the gain optimization allows us to go to the next smaller subaperture size, which gives us a larger region of correction

	N=32	***N=48, Opt***	***N=48***	***N=64, Opt***
SNR	4.89	2.16	2.16	1.19
MSE	.054	.074	.224	.210
Strehl	0.86	0.87	0.75	0.75

N=32 constant gain ~= N=48 optimized

- We can significantly increase detection area while maintaining correction level.



Averaged long-exposure PSFs of many random phase screens. PSFs same color scale

Computational requirements are satisfiable today

- FTR each timestep: $15N^2 \lg N + 20N^2$
- Estimating periodograms for t steps of telemetry:

$$N^2(4 + 2.5 \lg t)$$

- Averaging the periodograms and finding the optimal gain (k is for evaluations in root-finding):

$$N^2(1 + a + k) + 4k$$

- Assuming k = 10 (using fast method), a 64x64 system at 2.5k kHz has a maximum load of 1.42 GFLOPs/sec.

Optimal Modal FTR is our solution

- For an ExAOC system of 64x64 at 2.5kHz we can:
  - *use Optimal Modal FTR to reconstruct phase and optimize all 2050 gains in filter. Possibly update as fast as every 0.5 seconds*
  - *have nearly independent control of spatial locations in the PSF due to Fourier Frame.*
  - *get a significant improvement in performance on dim stars*
- We have end-to-end simulations of ExAOC case
- Analysis in Altair case shows FTR performs well in comparison to Altair matrix

Remaining to be addressed in CoDR (or wait for PDR?)

- Experimental tasks (MEMS issues at LAO)
  - *how to control actuators at the edge - most important issue*
  - *affect of dead or pegged actuators*
  - *verify response of MEMS and modal control measurement method*
- Theoretical tasks
  - *Lisa: use complex gains for control laws or other controller?*
  - *J.-P.: ??*
  - *Submit paper on Opt. Mod. FTR for peer review*