Effect of scintillation

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When an aberrated wave-front at height h above the telescope propagates to the telescope, the wave-front aberrations give rise to changes in the amplitude of the wave. Consider the atmosphere to be a single turbulent layer such that the C_n^2 profile consists of a delta function at h. Then, for an infinite aperture, it can be shown that the log-amplitude variance, σ_χ^2, is given by[1]

$$\sigma_\chi^2 = 0.563 k^{7/6} \int h^{5/6} C_n^2(h) \mathrm{d}h, \quad (1)$$

where $k = 2\pi/\lambda$ is the wavenumber. Using the relationship

$$r_0 = \left(0.423 k^2 \int C_N^2(h) \mathrm{d}h \right)^{-3/5} \quad (2)$$

we obtain the single-layer result

$$\sigma_\chi^2 = 0.288 \left(\sqrt{\lambda h} / r_0 \right)^{5/3}. \quad (3)$$

The reduction in Strehl due to scintillation is

$$S = \exp[-\sigma_\chi^2]. \quad (4)$$

Analytic calculations[1] and numerical simulations presented here both show that these result also holds for large astronomical telescopes. Random phase screens with Kolmogorov statistics were Fresnel propagated to the ground using Fresnel propagation code written specifically for Kolmogorov turbulence.[2]

As an example, consider the Mauna Kea turbulence profiles measured using SCIDAR over the nights of October 20-23, 2002.[3] Table 1 displays the worst turbulence profile from the point of view of scintillation over those four nights.

Height (km)	0.5	1	2	4	8	16	Total
$C_n^2 (1 \times 10^{-13} \mathrm{m}^{1/3})$	0.15	0.13	0.15	0.37	0.43	0.32	1.56

Table 1: The distribution of turbulence on October 21, 2002.[3]

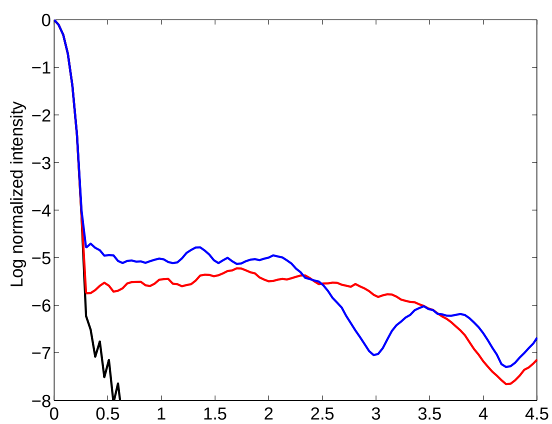
The value of r_0 at 500 nm is 0.246 m. Then an H-band image at 1.65 μm, would have a log amplitude variance of 0.0062, with a corresponding Strehl reduction of 0.62%. If the coherence length is halved, (*i.e.*, $r_0 = 0.123$), then the reduction in Strehl would be 1.96%. It can be concluded that the reduction in Strehl by scintillation is negligible when compared to the effect of wave-front aberrations.

Another question of interest is the contrast ratio as a function of spatial frequency. Since there is no obvious analytic solution to this problem, the answer was obtained by simulations. A two-dimensional square aperture was apodized using a Blackman window to remove the effect of diffraction. The Blackman window suppresses diffraction very aggressively, but significantly dimishes the effective aperture size. First, the effect of the different parameters was investigated using single phase screens and varying the height, turbulence strength and aperture diameter. The following relationships were obtained empirically between the contrast, $C(\theta)$, and the other parameters:

$$C(\theta) \propto D^2 r_0^{5/3} h^{-11/6}, \quad \theta \in [0, \sqrt{\lambda/h}]. \tag{5}$$

The dependence on pupil diameter, D, is to be expected. Likewise, the dependence on turbulence strength follows directly from Eq. (3). The most interesting result is that the contrast appears to be constant with angle up until half the angle corresponding to the Fresnel length, $\sqrt{\lambda z}$. Figure 1 shows the contrast due to scintillation for a diffraction-limited aperture, a phase screen at 4 km and a phase screen at 8 km. Here, $D = 4$ m and $r_0 = 0.2$ at 0.5 μm. It can be

Figure 1: Contrast for a phase screen at 4 km (red) and at 8 km (blue). The black line represents the diffraction-limited case.

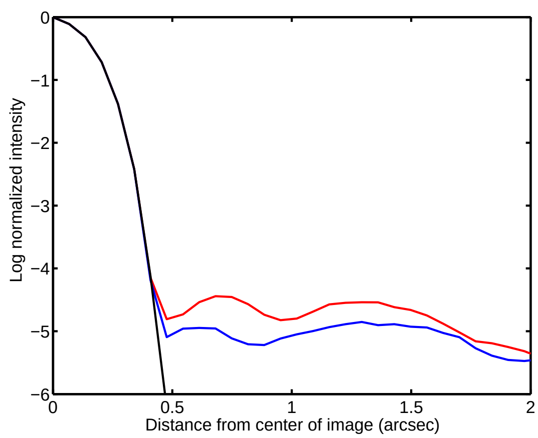


seen that the first null occurs at the angle corresponding to the Fresnel length.. The light that is scattered due to scintillation is scattered over a circle with radius $\sqrt{\lambda z}$. For the 4 km case this angle is $206265\sqrt{1.65 \times 10^{-6}/4000} = 4.2$

arcsec, which is still larger than highest spatial frequency that the DM can correct. Consequently, the contrast is inversely proportional to the amount of scattered light, $h^{-5/6}$, divided by the area over which the light falls, h. This means that the contrast reduction depends much more strongly on the height than the Strehl reduction and is dominated by the highest altitude turbulence layers.

Multi-layer simulations were subsequently run to verify the fact that the scintillation is additive. The complex amplitude was propagated from one phase screen to the next using Fresnel propagation. It was found that the total scintillation is equal to the scintillation contribution from each layer, according to Eq. (3). Due to limitations of computer memory, the simulation could only be run on apertures as large as 2.5 meters using only the layers at a height of 2 km and over. Using the turbulence profile of Table 1, we obtained the results plotted in Figure 2. For an 8-meter aperture, the contrast would be 10 times

Figure 2: Contrast for distributed turbulence using the turbulence profiles with the least (blue) and most (red) scintillation. The black line is the diffraction-limited contrast.



greater, (*i.e.*, $10^{5.6}$ for the case with the most scintillation presented).

References

[1] P. D. Stroud, "Diffraction and scintillation of laser beams by atmospheric turbulence," Los Alamos National Laboratory Note LAUR-93-1401 (1993).

[2] R. A. Johnston and R. G. Lane, "Modeling Scintillation from an Aperiodic Kolmogorov Phase Screen," Applied Optics **39**, 4761-4769 (2000).

[3] A. Tokovinin, "SCIDAR-MASS comparison," Internal report, www.ctio.noao.edu/ atokovin/profiler/compar.pdf (2004).